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A locally supersymmetric SU(6) grand unified theory without fine tuning and strong CP problems

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ABSTRACT

We construct a realistic supersymmetric SU(6) grand unified theory with a natural solution of the fine tuning and the strong CP problems. The prediction for $\sin^2\theta_W$ in one loop order is in good agreement with experiment.

I. INTRODUCTION

Supersymmetric grand unified theories are of interest at present, since they can provide a partial solution to the gauge hierarchy problem[1]. If we fine tune the parameters of the Lagrangian at the tree level, it is not destroyed by radiative corrections. Hence, once a large mass hierarchy is established at the tree level, it is stable under radiative corrections[2]. Supersymmetry, by itself, however, does not tell us why such a mass hierarchy should be there at the tree level. In particular, we still need fine tuning of parameters to one part in 10^{16} at the tree level, to keep the mass of the weak doublet higgs small compared to its color triplet partner. One solution to this problem is offered by the missing partner mechanism[3], in which the mass term for the doublet higgs is absent because of group theoretic reason. In a previous paper[4] we proposed another natural solution of this fine tuning problem in the context of locally supersymmetric grand unified theories. We also showed how to introduce a Peccei-Quinn symmetry in this type of models, which is spontaneously broken at 10^{10}GeV. In this paper we propose a simple realistic locally supersymmetric SU(6) grand unified theory based on the ideas developed in ref.4. One loop contribution to $\sin^2\theta_W$ in this model is in very good agreement with experiment. Also the constraint of perturbative unification (i.e. that the gauge coupling constant at the grand unification scale is smaller than unity) almost uniquely forces us to the minimal model of this kind.

In Sec.II of this paper we shall describe the model. In Sec.III we shall study the effect of one loop radiative corrections and show how this model provides a natural solution to the fine tuning and the strong CP problems. The SU(6) symmetry is spontaneously broken down to SU(3)×SU(3) ×U(1) at a scale of order 10^{17}GeV by the vev of an adjoint higgs. The SU(3)×SU(3)×U(1) symmetry, as well as the Peccei-Quinn symmetry is broken at a scale of order 10^{10}GeV by the vev of the fundamental higgs, the unbroken gauge group below 10^{10}GeV being SU(3)×SU(2)×U(1). This symmetry is broken to SU(3)×U(1) at a scale of order 10^{3}GeV by the vev of fundamental higgs due to radiative corrections. In Sec.IV we shall study the renormalization group equations for various gauge coupling constants and compute the value of $\sin^2\theta_w$, grand unification scale, and the value of the gauge coupling constant at the grand unification scale. In Sec. V we summarize our results, and suggest possible alterations of this model.

II. THE MODEL

The model constists of a superfield $\hat{\Phi}$ belonging to the adjoint(35) representation of SU(6), n+1 pairs of superfields $\hat{R}$, $\hat{\tilde{R}}$, $\hat{H}^{(i)}$, $\hat{\tilde{H}}^{(i)}$, (i=1,...n) belonging to the 6 and $\overline{6}$ representations respectively, and several singlet superfields $\hat{\Phi}_0$, $\hat{\sigma}$, $\hat{S}^{(i)}$ (i=1,...n). Besides these, there are three generations of quark lepton fields, each generation containing a $\overline{6}$, $\overline{6}$, 15 and 20 representations which we denote by $\hat{Q}^{(1)}_{\overline{6},s}$, $\hat{Q}^{(2)}_{\overline{6},s}$, $\hat{Q}_{15,s}$ and $\hat{Q}_{20,s}$ (s=1,..3) respectively. The superpotential is,

$$W = \lambda_1 \hat{\Phi}^3 + \lambda_2 \hat{\Phi}_0 \hat{\Phi}^2 + M_1 \hat{\Phi}^2 + M_2^2 \hat{\Phi}_0 + M_3 R\tilde{R}$$
$$+\sum_{i=1}^{n} (\alpha_1^{(i)} \hat{\Phi} \hat{H}^{(i)} \hat{\tilde{H}}^{(i)} + \alpha_2^{(i)} \hat{\Phi}_0 \hat{H}^{(i)} \hat{\tilde{H}}^{(i)} + \alpha_3^{(i)} \hat{S}^{(i)} \hat{H}^{(i)} \hat{\tilde{H}}^{(i)})$$
$$+ \beta_1 \hat{\sigma} \hat{R}\hat{\tilde{R}} + \beta_2 \hat{\sigma}^3 + \sum_{i=1}^{n} \sum_{s,t=1}^{3} \{ \gamma_{1,st}^{(i)} \hat{Q}^{(1)}_{\overline{6},s} \hat{Q}_{15,t} \hat{\tilde{H}}^{(i)}$$
$$+ \gamma_{2,st}^{(i)} \hat{Q}^{(2)}_{\overline{6},s} \hat{Q}_{15,t} \hat{\tilde{H}}^{(i)} + \gamma_{3,st}^{(i)} \hat{Q}_{15,s} \hat{Q}_{20,t} \hat{H}^{(i)}\} + \sum_{s,t=1}^{3} \gamma_{4,st} \hat{\sigma} Q_{20,s} Q_{20,t} \tag{1}$$

where the mass parameters M_i are of order 10^{16}-10^{17}GeV. For convenience of notation, we have dropped all the SU(6) indices in the above expression. In the absence of any supersymmetry breaking terms, the potential V, corresponding to the above superpotential, is given by,

$$V = \sum_i |F_{y_i}|^2 + \frac{1}{2} \sum_a |\sum_i y_i^+ T_a y_i|^2 \tag{2}$$

$$F_{y_i} = \partial W / \partial y_i \tag{3}$$

where the sum over i runs over the scalar components y_i of all the superfields appearing in (1), and the sum over α runs over all the generators of the gauge group. V has a supersymmetric minimum (V=0) at,

$$\langle \Phi \rangle = a_1 M \begin{pmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & -1 & & \\ & & & & -1 & \\ & & & & & -1 \end{pmatrix} \qquad \langle \Phi_0 \rangle = a_2 M \tag{4}$$

$$\langle \text{ALL OTHER FIELDS} \rangle = 0$$

except the vev of $S^{(i)}$, which is undetermined at this stage. Here M is a mass parameter of the order of M_1, M_2 and M_3, and a_1 and a_2 are constants of order unity. This breaks the SU(6) gauge group to SU(3)×SU(3)×U(1) at a scale of order 10^{16}-10^{17}GeV. The fields R, $\tilde{R}$, $H^{(i)}$, $\tilde{H}^{(i)}$, in general, acquire masses of order M, unless $\langle S^{(i)} \rangle$ takes special value which exactly cancels the mass terms of either the upper three or the lower three components of $H^{(i)}$, $\tilde{H}^{(i)}$. The fields σ, $S^{(i)}$ and $Q^{(i)}_{6,s}$, $Q_{15,s}$ and $Q_{20,s}$ remain massless at this stage.

Let us now consider the effect of supersymmetry breaking. Supersymmetry is assumed to be spontaneously broken by the superhiggs mechanism[5] at a scale of order 10^{11}GeV, giving the gravitino a mass $m_g \sim 10^2$-10^3GeV. But the supersymmetry breaking takes place entirely in the 'hidden sector', which couples to the observable sector (containing all the fields that appear in Eq.(1)) only through the effect of gravity[6]. The net effect of supersymmetry breaking on the observable sector is to introduce explicit soft supersymmetry breaking in the Lagrangian of the form[6],

$$\{m_g (A-3)\, W(y) + \sum_i m_g\, y_i\, \partial W/\partial y_i + h.c.\} - m_g^2 \sum_i |y_i|^2 \tag{5}$$

where y_i denote the scalar components of all the superfields which appear in (1), and A is a constant of order unity, whose precise value depends on the underlying supergravity theory. These terms may be expressed in terms of the superfields as follows:

$$\int d^2\theta \left\{ \eta\, (A-3)\, W(\hat{y}) + \eta \sum_i \hat{y}_i \frac{\partial W(\hat{y})}{\partial \hat{y}_i} + h.c. \right\}$$
$$- \int d^2\theta\, d^2\bar{\theta}\;\; \bar{\eta}\, \eta \sum_i \bar{y}_i\, y_i \tag{6}$$

where,

$$\eta = m_g\, \theta^2 \tag{7}$$

is a spurion superfield.

Let us now try to minimize the potential including these new terms. For simplicity of discussion we shall drop the σ, Q, R and $\tilde{R}$ fields from our discussion, since the inclusion of these fields do not change any of the results that will be discussed below. The new potential, which is obtained by subtracting (5) from the potential given in (2), may be written as,

$$\tilde{V} = \sum_i \left| \frac{\partial W}{\partial y_i} - m_g y_i^* \right|^2 - \left(m_g (A-3) W(y) + h.c. \right)$$

$$+ \frac{1}{2} \sum_a \left| \sum_i y_i^\dagger T_a y_i \right|^2 \tag{8}$$

The minimum of the potential satisfies the condition,

$$\frac{\partial W}{\partial y_i} - m_g y_i^* \lesssim m_g^2 \tag{9}$$

if y_i is either of the heavy fields Φ or Φ_0. This may be illustrated by considering a simple model containing only one field y with mass and vev of order M. The minimization of the potential then requires that,

$$\left(\frac{\partial^2 W}{\partial y^2} - m_g \right) \left(\frac{\partial W}{\partial y} - m_g y \right) = (A-3)\, m_g \frac{\partial W}{\partial y} \tag{10}$$

where we have assumed $\langle y \rangle$ to be real for simplicity, and ignored the contribution from the D term, since in the present case it vanishes automatically if, for example $\langle \Phi \rangle$ is real, and $\langle H^{(i)} \rangle$ and $\langle \tilde{H}^{(i)} \rangle$ are equal. We may solve equation (10) iteratively, keeping in mind that $\partial W/\partial y$ vanishes in the supersymmetric limit. If we start at the point where $\partial W/\partial y$ vanishes, then after the first iteration we get $\partial W/\partial y = m_g y$. We may substitute this value of $\partial W/\partial y$ on the right hand side of Eq. (10), to get,

$$\left(\frac{\partial W}{\partial y} - m_g y\right)_{new} = (A-3)\, m_g^2\, y / \left(\frac{\partial^2 W}{\partial y^2} - m_g\right) \sim m_g^2 \tag{11}$$

since y and $\partial^2 W/\partial y^2$ are both of order M. We may substitute the new value of $\partial W/\partial y$ in the right hand of side of Eq. (10) to get back Eq. (9) again. This result may easily be generalized to the case of more than one heavy field to get Eq. (9).

Let us now turn to the fields $H^{(i)}$, $\tilde{H}^{(i)}$ and $S^{(i)}$. The vev of the fields $H^{(i)}$ and $\tilde{H}^{(i)}$ vanished in the supersymmetric limit, hence there certainly exists a local minimum of the potential at vanishing vev of these fields. (Since $\partial W/\partial y$ and $m_g y$ both vanish at this point, the derivatives of $\tilde{V}$ with respect to these fields vanish at this point.) The vev's of $S^{(i)}$ were undetermined in the exactly supersymmetric limit, i.e. the derivative of the potential with respect to $S^{(i)}$ vanished for all values of $S^{(i)}$. The derivative of the new potential $\tilde{V}$ with respect to $S^{(i)}$ vanishes at $S^{(i)}=0$, but not at other values of $S^{(i)}$. Hence the potential $\tilde{V}$ has a local minimum at $\langle H^{(i)}\rangle = \langle \tilde{H}^{(i)}\rangle = \langle S^{(i)}\rangle = 0$. The fact that $\partial W/\partial\Phi$ and $\partial W/\partial\Phi_0$ are equal to $m_g\Phi^*$ and $m_g\Phi_0^*$ respsctively, instead of being zero, causes a shift of order m_g in the vev of Φ and Φ_0 from their values in the supersymmetric limit.

There is however, another local minimum, which is of interest to us. To see this minimum let us write down the full potential, ignoring the σ, R, $\tilde{R}$ and Q fields.

$$
\begin{aligned}
\tilde{V} = & \left| \frac{\partial W}{\partial \Phi} - m_g \Phi^* \right|^2 + \left| \frac{\partial W}{\partial \Phi_0} - m_g \Phi_0^* \right|^2 \\
& + \sum_{i=1}^{n} \left| \left(\alpha_1^{(i)} \Phi + \alpha_2^{(i)} \Phi_0 + \alpha_3^{(i)} S^{(i)}\right) H^{(i)} - m_g \tilde{H}^{(i)*} \right|^2 \\
& + \sum_{i=1}^{n} \left| \tilde{H}^{(i)} \left(\alpha_1^{(i)} \Phi + \alpha_2^{(i)} \Phi_0 + \alpha_3^{(i)} S^{(i)}\right) - m_g H^{(i)*} \right|^2 \\
& + \sum_{i=1}^{n} \left| \alpha_3^{(i)} H^{(i)} \tilde{H}^{(i)} - m_g S^{(i)*} \right|^2 - m_g (A-3) \left(\lambda_1 \Phi^3 + \lambda_2 \Phi_0 \Phi^2 \right. \\
& \left. + M_1 \Phi^2 + M_2^2 \Phi_0 \right) - m_g (A-3) \sum_{i=1}^{n} \tilde{H}^{(i)} \left(\alpha_1^{(i)} \Phi + \alpha_2^{(i)} \Phi_0 + \alpha_3^{(i)} S^{(i)}\right) H^{(i)} \\
& + \frac{1}{2} \sum_a \left| \sum_i y_i^{+} T_a y_i \right|^2
\end{aligned}
\tag{12}
$$

In the new local minimum, $S^{(i)}$ takes a vev of order M, so as to keep either the upper three components or the lower three components of $H^{(i)}$, $\tilde{H}^{(i)}$ massless. For definiteness we shall assume that it is the lower three components of $H^{(i)}$ which remain massless. Hence,

$$
\langle S^{(i)} \rangle = -\left(\alpha_1^{(i)} \Phi_{66} + \alpha_2^{(i)} \Phi_0\right) / \alpha_3^{(i)} + O(m_g) \tag{13}
$$

$H^{(i)}$, $\tilde{H}^{(i)}$ then acquire vev's of order $\sqrt{m_g M}$ so that,

$$
\left(\langle H^{(i)} \rangle \langle \tilde{H}^{(i)} \rangle \right)_{SINGLET} = m_g \langle S^{(i)} \rangle / \alpha_3^{(i)} + O(m_g^2) \tag{14}
$$

which makes the $\partial W/\partial S^{(i)} - m_g S^{(i)*}$ term small. The $O(m_g)$ and $O(m_g^2)$ terms in $\langle S^{(i)}\rangle$ and $\langle H^{(i)}\rangle\langle\tilde{H}^{(i)}\rangle$ respectively are due to the $m_g(A-3)W(y)$ term in $\tilde{V}$. Φ and Φ_0 adjusts themselves so as to make $\partial W/\partial\Phi - m_g\Phi^*$ and $\partial W/\partial\Phi_0 - m_g\Phi_0^*$ terms to be of order m_g^2. Since now $\partial W/\partial\Phi$ and $\partial W/\partial\Phi_0$ terms receive extra contribuions of order $m_g M$ from the $H^{(i)}\tilde{H}^{(i)}$ terms, the vev of Φ and Φ_0 are shifted by order m_g from the previous minimum.

We may now estimate the difference in energy density between the two minima. One source of difference is the

$$-m_g(A-3)(\lambda_1\Phi^3 + \lambda_2\Phi_0\Phi^2 + M_1\Phi^2 + M_2^2\Phi_0)$$

$$\equiv -m_g(A-3)\,W_0(\Phi,\Phi_0) \tag{15}$$

term in the potential. If $\Delta\Phi$ and $\Delta\Phi_0$ denote the difference in the values of Φ and Φ_0 respectively in the two minima, the contribution from (15) to the difference in the energy density between the two minima is,

$$-m_g(A-3)\left\{\frac{\partial W_0}{\partial\Phi_0}\Delta\Phi_0 + \frac{\partial W_0}{\partial\Phi}\Delta\Phi\right\}$$

Since $\partial W_0/\partial\Phi$ and $\partial W_0/\partial\Phi_0$ are of order $m_g M$, and $\Delta\Phi$ and $\Delta\Phi_0$ are of order m_g, this contribution goes as $m_g{}^3 M$.

Other major sources of the difference in energy density between the two minima are the $|F_{H^{(i)}} - m_g H^{(i)*}|^2$, $|F_{\tilde{H}^{(i)}} - m_g\tilde{H}^{(i)*}|^2$, and the $-m_g(A-3)(W(y)-W_0(y))$ term in $\tilde{V}$. Each of these contributions is again of order $m_g{}^3 M$. Thus the total difference in energy between the two minima is of order $m_g{}^3M$. We do not attempt to compute its exact value here, since, as we shall see in the next section, radiative corrections produce a much larger energy difference ($\sim m_g{}^2M^2$) between these two minima.

III. RADIATIVE CORRECTIONS

In this section we shall study the effect of radiative corrections to the potential discussed in Sec.II. In an exactly supersymmetric theory, if supersymmetry is unbroken, the only effect of radiative corrections is to produce wave-function renormalization of various fields in the superpotential. However, due to the presence of the explicit supersymmetry breaking terms given in (6) there will be higher loop radiative corrections to the effective action of the form[7],

$$\int d^2\theta \, d^2\bar{\theta} \; f\,(\hat{y}_i\,,\ \hat{\bar{y}}_i\,,\ \eta,\ \bar{\eta}\,) \tag{17}$$

where the function f is a polynomial in the superfields $\hat{y}_i$, their covariant derivatives, and the spurion superfield η. It was pointed out by various authors[8], that the presence of these terms may produce masses and vev's of order $\sqrt{m_q M}$ of the fields which had zero mass and/or vacuum expectation value in the exactly supersymmetric limit. In this particular model, the important radiatively induced terms in the effective action, which produce such effects, are of the form,

$$\begin{aligned}&\sum_{i=1}^{n}\left(m_g\, F^{*}_{S^{(i)}}\; f^{(i)}(\Phi, \Phi_0, S^{(i)}, M) + h.c.\right)\\ &+\sum_{i=1}^{n}\left(m_g^2\, S^{(i)*}\; g^{(i)}(\Phi, \Phi_0, S^{(i)}, M) + h.c.\right)\end{aligned} \tag{18}$$

where the functions f and g are of order M times logarithmically divergent functions. Typical diagrams, contributing to the functions $f^{(i)}$ and $g^{(i)}$ are shown in Fig.1. Similar terms involving the field σ are also generated, but the effect of those terms will be discussed later. There are also radiative corrections involving F_{Φ} and F_{Φ_0}, but we ignore them in our discussion, since they do not qualitatively change any of the results discussed below.

We may now eliminate the F components of various fields by using the equations of motion. The F components of the other fields are given by Eq.(3), except $F_{S^{(i)}}$, which is given by,

$$F^*_{S^{(i)}} = \alpha_3^{(i)} H^{(i)} \tilde{H}^{(i)} + m_g f^{(i)*} \tag{19}$$

The part of the potential, containing the Φ, Φ_0, $S^{(i)}$, $H^{(i)}$ and $\tilde{H}^{(i)}$ fields is given by,

$$\begin{aligned}
&\left|\frac{\partial W}{\partial \Phi} - m_g \Phi^*\right|^2 + \left|\frac{\partial W}{\partial \Phi_0} - m_g \Phi_0^*\right|^2 \\
&+ \sum_{i=1}^{n} \left\{ \left|\frac{\partial W}{\partial H^{(i)}} - m_g H^{(i)*}\right|^2 + \left|\frac{\partial W}{\partial \tilde{H}^{(i)}} - m_g \tilde{H}^{(i)*}\right|^2 \right. \\
&\left. + \left|\frac{\partial W}{\partial S^{(i)}} + m_g f^{(i)*}\right|^2 - \left(m_g S^{(i)} \frac{\partial W}{\partial S^{(i)}} + h.c.\right) + m_g^2 |S^{(i)}|^2 \right\} \\
&- \left(m_g (A-3) W + h.c.\right) - m_g^2 \sum_{i=1}^{n} \left(g^{(i)*} S^{(i)} + h.c.\right) \\
&+ \frac{1}{2} \sum_a \left| \sum_i y_i^+ T_a y_i \right|^2
\end{aligned} \tag{20}$$

In order to minimize the potential, it is more convenient to write it as,

$$\left|\frac{\partial W}{\partial \Phi} - m_g \Phi^*\right|^2 + \left|\frac{\partial W}{\partial \Phi_0} - m_g \Phi_0^*\right|^2$$
$$+ \sum_{i=1}^{n} \left\{ \left|\frac{\partial W}{\partial H^{(i)}} - m_g H^{(i)*}\right|^2 + \left|\frac{\partial W}{\partial \tilde{H}^{(i)*}} - m_g \tilde{H}^{(i)*}\right|^2 \right.$$
$$\left. + \left|\frac{\partial W}{\partial S^{(i)}} + m_g f^{(i)*} - m_g S^{(i)*}\right|^2 \right\}$$
$$-\left(m_g (A-3)\; W(y) + h.c.\right) + m_g^2 \sum_{i=1}^{n} \left\{ (f^{(i)*} - g^{(i)*})\, S^{(i)} + h.c. \right\}$$
$$+ \frac{1}{2} \sum_{a} \left| \sum_{i} y_i^\dagger T_a y_i \right|^2 \tag{21}$$

We may minimize the potential, remembering that the functions $f^{(i)}$, $g^{(i)}$ are of order M. There are two different kinds of local minimum of the potential. In the first kind, $\langle H^{(i)}\rangle$, $\langle \tilde{H}^{(i)}\rangle$ are zero, and $\langle S^{(i)}\rangle$ is determined by minimizing,

$$\sum_{i} \left[\left| m_g (f^{(i)*} - S^{(i)*}) \right|^2 + m_g^2 \left\{ (f^{(i)*} - g^{(i)*})\, S^{(i)} + h.c. \right\} \right. \tag{22}$$

which produces a vev of $S^{(i)}$ of order M. In the second class of solutions, $S^{(i)}$ takes a vev so that either the lower three components or the upper three components of $H^{(i)}$, $\tilde{H}^{(i)}$ remains massless. For definiteness, we shall assume that the lower three components remain massless. (i.e. $S^{(i)}$ takes the value given in (13)). $H^{(i)}$, $\tilde{H}^{(i)}$ then acquire vev of the form,

$$\mathcal{H}^{(i)} = \tilde{\mathcal{H}}^{(i)} \simeq \sqrt{\frac{m_g\,(S^{(i)*} - f^{(i)*})}{\alpha_3^{(i)}}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \tag{23}$$

so as to make the $|F_{S^{(i)}}|^2$ term to be of order m_g^2.

Which of these two types of solutions has the lower energy depends on the parameters of the superpotential W. We shall, however, assume that for each i, it is the second type of solution which has the lowest energy. (Note that the difference in energy density between the two vacua is of order $m_g{}^2M^2$, because of the $m_g{}^2(f^{(i)}-g^{(i)})S^{(i)*}$ term.) This breaks the $SU(3)\times SU(3)\times U(1)$ symmetry to $SU(3)\times SU(2)\times U(1)$ at a scale of order $\sqrt{m_g M}\sim 10^{10}$GeV, since $f_1^{(i)}\sim M$, and $\alpha_3^{(i)}\sim 1$.

There is, however, one subtle point which is worth mentioning. It may seem that the potential given in (21) is independent of the relative directions of various $H^{(i)}$'s, so that the fields $H^{(i)}$, $\tilde{H}^{(i)}$ could take vev of the form,

$$\mathcal{H}^{(1)} = \tilde{\mathcal{H}}^{(1)} = \sqrt{\frac{m_g\,(S^{(1)*} - f^{(1)*})}{\alpha_3^{(1)}}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \qquad \mathcal{H}^{(2)} = \tilde{\mathcal{H}}^{(2)} = \sqrt{\frac{m_g\,(S^{(2)*} - f^{(2)*})}{\alpha_3^{(2)}}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \sqrt{x} \\ \sqrt{1-x} \end{pmatrix} \tag{24}$$

$$0 \le x \le 1$$

The value of the potential, as given in (20) and (21), is independent of x if we do not consider the $F_\Phi - m_g\Phi^*$ and $F_{\Phi_0} - m_g\Phi_0^*$ terms. If x=0, the unbroken symmetry group below 10^{10}GeV is SU(3)×SU(2)×U(1), whereas, if x~1, the unbroken symmetry group below 10^{10}GeV is SU(3)×U(1). The degeneracy, however, is removed when we take into account the correction to $\langle\Phi\rangle$ due to the vev of $H^{(i)}$, $\tilde{H}^{(i)}$. To see this, consider the $\partial W/\partial\Phi - m_g\Phi^*$ term appearing in (21), which is given by,

$$3\lambda_1(\Phi^2)_{35} + \lambda_2\Phi_0\Phi + M_1\Phi + \sum_{i=1}^{n}\alpha_1^{(i)}H^{(i)}\tilde{H}^{(i)} - m_g\Phi^* \tag{25}$$

Now suppose $H^{(1)}$, $\tilde{H}^{(1)}$ acquire vev of the form given in (12). In order to minimize the potential, we need to make small shifts $\delta\langle\Phi\rangle$ and $\delta\langle\Phi_0\rangle$ in $\langle\Phi\rangle$ and $\langle\Phi_0\rangle$ of the form,

$$\delta\langle\Phi_0\rangle = b_1 m_g \qquad \delta\langle\Phi\rangle = m_g\begin{pmatrix} b_2 & & & & & \\ & b_2 & & & & \\ & & b_2 & & & \\ & & & b_3 & & \\ & & & & b_3 & \\ & & & & & -3b_2-2b_3 \end{pmatrix} \tag{26}$$

If now $S^{(2)}$ adjusts itself so as to make the mass of $H_6^{(2)}$, $\tilde{H}_6^{(2)}$ vanish, the mass of $H_m^{(2)}$, $\tilde{H}_m^{(2)}$ (m=4,5) is of order m_g, and in order to minimize the potential, the x of Eq.(12) must be zero. Thus the unbroken subgroup below 10^{10}GeV is SU(3)×SU(2)×U(1). Alternatively, $S^{(2)}$ may adjust itself so as to make the mass of $H_m^{(2)}$, $\tilde{H}_m^{(2)}$ (m=4,5) vanish. Then the x in Eq.(12) must be 1, and the unbroken symmetry group below 10^{10}GeV is SU(3)×U(1). Which of these two minima has the lowest energy depends on the values of various parameters. We shall assume that the SU(3)×SU(2)×U(1) symmetric ground state is the state of lowest energy below 10^{10}GeV.

As we have already seen, the vev of $S^{(i)}$ which keeps the 6th component of $H^{(i)}$ and $\tilde{H}^{(i)}$ massless, also keeps the mass of the fourth and the fifth components of $H^{(i)}$ and $\tilde{H}^{(i)}$ to be of order m_g. Thus we get n pairs of low mass ($\sim m_g$) weak doublet Higgs. Of these, one particular linear combination is absorbed by the gauge bosons corresponding to the broken generators of the SU(3)×SU(3)×U(1) group through Higgs mechanism. Another linear combination acquires a mass of order 10^{10}GeV through the D terms of the potential, and becomes the part of a complete massive vector supermultiplet. We are then left with (n-1) pairs of weak doublet Higgs of mass $\sim m_g$. Some of these masses may be driven to be negative due to radiative corrections[6], thus producing a spontaneous breakdown of the $SU(2)^W\times U(1)$ symmetry at a scale of order $m_g \sim 10^3$ GeV.

Let us now turn to the σ field. Due to its coupling to the heavy field R, $\tilde{R}$, one loop radiative corrections generate terms in the potential of the form,

$$a_1 m_g F_\sigma^* M_3 + b_1 m_g^2 \sigma^* M_3 + h.c. \tag{27}$$

If we eliminate the F_σ term from the potential, the effective potential, involving the σ field is given by,

$$|3\beta_2 \sigma^2 + a_1^* m_g M_3^*|^2 + (\beta_2 A m_g \sigma^3 + b_1 m_g^2 \sigma^* M_3 + h.c.)$$

$$+ m_g^2 |\sigma|^2 \tag{28}$$

where we have set the vev's of the R, $\tilde{R}$ and the Q fields to be zero. The potential has a minimum at,

$$\sigma \simeq \sqrt{-a_1^* m_g M_3^*/3\beta_2} \sim \sqrt{m_g M_3} \tag{29}$$

The quarks get mass in the same way as mentioned in Ref.4. If we decompose the quark content of the theory in terms of representations of the SU(5) subgroup of SU(6), then three linear combinations of $Q^{(k)}_{\bar{5}(\bar{6}),s}$, (s=1,2,3; k=1,2) (here $Q^{(k)}_{\bar{5}(\bar{6})}$ denote the part of $Q^{(k)}_{\bar{6}}$ which transform as the $\bar{5}$ component of SU(5)) will combine with the three $Q_{5(15),s}$ to get a mass of order $\langle \tilde{H}^{(i)}_{6} \rangle$. Three orthogonal linear combinations $(Q^{Phys.}_{\bar{5},s})$ of $Q^{(k)}_{\bar{5}(\bar{6}),s}$ remain massless at this stage. Similarly, three particular linear combinations of $Q_{10(15),s}$ and $Q_{10(20),s}$ combine with the three $Q_{\overline{10}(20),s}$ to get masses of order $\langle\sigma\rangle$ or $\langle H_6^{(i)}\rangle$. Three orthogonal linear combinations ($Q_{10,s}^{\text{Phys.}}$) of $Q_{10(15),s}$ and $Q_{10(20),s}$ remain massless at this stage. Also, $Q_{1(\bar{6}),s}^{(k)}$ (k=1,2; s=1,2,3) remain massless. After the breakdown of the $SU(2)^W \times U(1)$ symmetry, $Q_{\bar{5}}^{\text{Phys.}}$ and $Q_{10}^{\text{Phys.}}$ acquire masses of the form,

$$Q_{\bar{5}}^{\text{Phys.}} \, Q_{10}^{\text{Phys.}} \, \tilde{H}_5^{(i)} \quad \text{and} \quad Q_{10}^{\text{Phys.}} \, Q_{10}^{\text{Phys.}} \, H_5^{(i)}$$

thus producing the usual low energy spctrum of fermions. Besides the usual fermions, there are two massless $SU(3) \times SU(2) \times U(1)$ singlets per generation.

It is easy to introduce a Peccei-Quinn symmetry[9] in this model. For example, let us consider a theory with two pairs of higgses with the following coupling to the quark lepton fields:

$$\sum_{s,t=1}^{3} \Big\{ \gamma_{1,st} \, Q_{\bar{6},s}^{(1)} \, Q_{15,t} \, \tilde{H}^{(1)} + \gamma_{2,st} \, Q_{\bar{6},s}^{(2)} \, Q_{15,t} \, \tilde{H}^{(2)}$$
$$+ \gamma_{3,st} \, Q_{15,s} \, Q_{20,t} \, H^{(2)} + \gamma_{4,st} \, Q_{20,s} \, Q_{20,t} \, \sigma \Big\} \tag{30}$$

The model then has a Peccei-Quinn symmetry,

$$H^{(1)} \to e^{i\theta} H^{(1)}, \quad H^{(2)} \to e^{-i\theta} H^{(2)}, \quad \tilde{H}^{(1)} \to e^{-i\theta} \tilde{H}^{(1)}, \quad \tilde{H}^{(2)} \to e^{i\theta} \tilde{H}^{(2)}$$

$$Q_{15,s} \to e^{i\theta} Q_{15,s}, \quad Q_{\bar{6},s}^{(1)} \to Q_{\bar{6},s}^{(1)}, \quad Q_{\bar{6},s}^{(2)} \to e^{-2i\theta} Q_{\bar{6},s}^{(2)} \tag{31}$$

while all the other fields remain unchanged under this transformation. This symmetry is broken spontaneously at a scale of order 10^{10}GeV by the vev of $H^{(i)}_{6}$, $\tilde{H}^{(i)}_{6}$, thus giving rise to an invisible axion with decay constant of order 10^{10}GeV. This falls within the narrow range of values allowed by the present cosmology[10].

The color triplet partner of the weak doublet higgses acquire masses of order $M \sim 10^{16}$-10^{17}GeV in this model. Thus, if we want cosmological baryon production at a temperature of order 10^{10}GeV[11] we must introduce new color triplet fields. This may easily be done by introducing a pair $(6, \bar{6})$ of higgs superfields $\hat{H}$, $\hat{\tilde{H}}$ with the coupling,

$$\delta_1 \hat{\sigma} \hat{\tilde{H}} \hat{H} + \sum_{s,t} \delta_{2,st} Q^{(i)}_{\bar{6},s} Q_{15,t} \tilde{H} \qquad (32)$$

The vev ($\sim 10^{10}$GeV) of σ produces a mass of order 10^{10}GeV for all components of H. The baryon number may then be generated at a temperature of order 10^{10}GeV through the decay of the higgs particle $\tilde{H}$, and also the decay of the heavy fermions through the intermediate higgs exchange. The complex phase in the decay amplitude may be generated due to the Kobayshi-Maskawa type phases, arising from the mass matrix of the heavy Q fields.

IV. EVOLUTION OF THE GAUGE COUPLING CONSTANTS

The calculation of $\sin^2\theta_w$ in this model is slightly more tricky than that in the usual SU(5) models, since there is an intermediate mass scale corresponding to the SU(3)×SU(3)×U(1) symmetry breaking scale v. Let g_3, g_2 and g_1 denote the coupling constants for the $SU(3)^c$, $SU(2)^W$ and U(1) gauge groups respectively, below the scale v. One loop contribution to the renormalization group equations give us the following evolution equations for $m_w<\mu_1,\mu_2<v$,

$$4\pi\left\{g_3(\mu_1)^{-2}-g_3(\mu_2)^{-2}\right\}=(2\pi)^{-1}\left(\ln\frac{\mu_1}{\mu_2}\right)\left(9-\tfrac{1}{2}N_{\bar 5}-\tfrac{3}{2}N_{10}\right)$$

$$4\pi\left\{g_2(\mu_1)^{-2}-g_2(\mu_2)^{-2}\right\}=(2\pi)^{-1}\left(\ln\frac{\mu_1}{\mu_2}\right)\left(6-\tfrac{1}{2}N_{\bar 5}-\tfrac{3}{2}N_{10}-\tfrac{1}{2}H\right)$$

$$4\pi\left\{g_1(\mu_1)^{-2}-g_1(\mu_2)^{-2}\right\}=(2\pi)^{-1}\left(\ln\frac{\mu_1}{\mu_2}\right)\left(-\tfrac{1}{2}N_{\bar 5}-\tfrac{3}{2}N_{10}-\tfrac{3}{10}H\right) \tag{33}$$

where $N_{\bar 5}$ and N_{10} are the number of Q fields belonging to the $\bar 5$ and 10 representations of SU(5) with mass of order m_g or less. [We find it more convenient to state the result in terms of SU(5) multiplets, since in the range $m_w<\mu_i<v$, the heavy and the light fields fall into full SU(5) multiplets.] H is the number of light higgs doublet fields. We have

assumed that in this range there is no light color triplet higgs.

We may use Eqs.(33) to relate the values of g_1, g_2 and g_3 at a scale m_w to those at the scale v. Above the scale v the unbroken group is SU(3)×SU(3)×U(1). In this, the first SU(3) subgroup is identical to the color subgroup, hence we may identify its coupling constant to g_3 at v. The second SU(3) subgroup contains $SU(2)^{weak}$ as a subgroup. Hence if we denote its coupling constant by $\tilde{g}_3$, we may write,

$$\tilde{g}_3(v) = g_2(v) \qquad (34)$$

The calculation of the coupling constant $\tilde{g}_1$ of the U(1) subgroup of SU(3)×SU(3)×U(1) in terms of the coupling constants of the SU(3)×SU(2)×U(1) subgroup needs some work. Let T_1' denote the generator of the U(1) subgroup of SU(3)×SU(2)×U(1), T_1 denote the generator of the U(1) subgroup of SU(3)×SU(3)×U(1), and T_2 denote the hypercharge generator of the second SU(3) subgroup of SU(3)×SU(3)×U(1), all normalized to $Tr(T_aT_b)=\delta_{ab}/2$. Then we may write,

$$T_1' = \sqrt{\frac{4}{5}}\, T_1 - \sqrt{\frac{1}{5}}\, T_2 \qquad (35)$$

Remembering that T_1 couples with a coupling constant $\tilde{g}_1$, T_2 with a coupling constant $\tilde{g}_3$, and T_1' with a coupling constant g_1, we have the relation,

$$g_1^{-2} = \frac{4}{5}\, \tilde{g}_1^{-2} + \frac{1}{5}\, \tilde{g}_3^{-2} \qquad (36)$$

Hence,

$$\tilde{g}_1(v)^{-2} = \frac{5}{4}\, g_1(v)^{-2} - \frac{1}{4}\, g_2(v)^{-2} \tag{37}$$

Equations (34) and (37) help us in relating the coupling constants g_3, $\tilde{g}_3$ and $\tilde{g}_1$ of the SU(3)×SU(3)×U(1) subgroup to those of the SU(3)×SU(2)×U(1) subgroup. The evolution of these coupling constants in the range $v<\mu_1,\mu_2<M_{GUT}$ are governed by the following equations:

$$4\pi\left(g_3(\mu_1)^{-2} - g_3(\mu_2)^{-2} \right) = (2\pi)^{-1}\left(\ln\frac{\mu_1}{\mu_2}\right)\left(9 - \frac{N_{\bar{6}}}{2} - 2N_{15} - 3N_{20} - \frac{1}{2}T\right)$$

$$4\pi\left(\tilde{g}_3(\mu_1)^{-2} - \tilde{g}_3(\mu_2)^{-2} \right\} = (2\pi)^{-1}\left(\ln\frac{\mu_1}{\mu_2}\right)\left(9 - \frac{N_{\bar{6}}}{2} - 2N_{15} - 3N_{20} - \frac{1}{2}H\right)$$

$$4\pi\left(\tilde{g}_1(\mu_1)^{-2} - \tilde{g}_1(\mu_2)^{-2} \right\} = (2\pi)^{-1}\left(\ln\frac{\mu_1}{\mu_2}\right)\left(- \frac{N_{\bar{6}}}{2} - 2N_{15} - 3N_{20} - \frac{H}{4} - \frac{T}{4}\right)$$

$$\tag{38}$$

T being the number of colored higgs triplets with mass of order v.

Eqs. (33), (34), (37) and (38) give us the evolution equation for all the coupling constants from m_W to the grand unification scale. In these equations there are altogether three unknowns, $\sin^2\theta_W$ at m_W, the intermediate scale v, and the grand unification scale M_{GUT}. The constraint that all the three coupling constants meet at the grand unification scale give us two equations relating these three unknowns.

As a result, we can solve for $\sin^2\theta_w$ and M_{GUT} as a function of v. We consider the minimal model with two pairs of higgses $H^{(i)}$, $\tilde{H}^{(i)}$ (i=1,2). The addition of the extra pair H, $\tilde{H}$, all of whose components acquire the same mass, does not change the prediction for M_{GUT} or $\sin^2\theta_w$. Taking $\alpha^{-1}_{QCD}(m_w)=9.9$ and $\alpha^{-1}_{e.m.}(m_w)=127.56$, we get the following values of $\sin^2\theta_w$ and M_{GUT} for different values of v:

v	$\sin^2\theta_w$	M_{GUT}
$10^7\ m_w$	.206	$10^{15.5}\ m_w$
$10^8\ m_w$	.211	$10^{15.25}\ m_w$
$10^9\ m_w$	.216	$10^{15}\ m_w$
$10^{10}\ m_w$	.220	$10^{14.75}\ m_w$

Thus we see that the value of $\sin^2\theta_w$, as well as the value of M_{GUT} is relatively insensitive to the value of v. The best value is obtained for $v \sim 10^{11}$GeV. The corresponding GUT scale is of order 10^{17} GeV. Taking $m_g \sim 10^3$GeV, we see that the relation $v=\sqrt{m_g M}$ is satisfied within a factor of 10. This may be obtained by adjusting various coupling constants.

We can also calculate the value of α_{GUT} in our theory, but its value is sensitive to the masses of various particles in the model. If we assume that $\langle\sigma\rangle \sim \langle H_6\rangle$, then α_{GUT} reaches the strong coupling limit before we reach the grand unification scale, for most values of v. On the other hand, if we consider $\langle\sigma\rangle$ to be one or two orders of magnitude higher than $\langle H_6\rangle$, then the situation is much better. For $\langle\sigma\rangle=10^{11}m_w$, we get the following values of α_{GUT} for different values of v:

v	α_{GUT}^{-1}
$10^7 m_w$	1.1
$10^8 m_w$	3.1
$10^9 m_w$	5.1
$10^{10} m_w$	7.1

Thus we see that for $v>10^{10}$GeV, α_{GUT} is still within the perturbative regime. The reader may wonder whether higher loop corrections may affect the value of $\sin^2\theta_w$, since α_{GUT} is not very small. However, α_{GUT} becomes large only very near the grand unification scale (for example for $v=10^9m_w$, $\langle\sigma\rangle=10^{11}m_w$, $\alpha^{-1}_{QCD}(M_{GUT}/10)=8.8$), hence we do not expect the higher loop corrections to affect the value of $\sin^2\theta_w$ appreciably.

The large vev of σ may be obtained by taking a small value of β_2 and large value of M_3 in the superpotential (1). An alternative possibility will be mentioned in the next section.

V. SUMMARY AND DISCUSSIONS

In this paper we have proposed a locally supersymmetric grand unified theory based on the SU(6) gauge group with a natural solution of the fine tuning problem. This model also has a Peccei-Quinn symmetry, which is spontaneously broken at a scale of order 10^{10}-10^{11}GeV, thus giving rise to a harmless invisible axion. The SU(6) symmetry is spontaneously broken down to SU(3)×SU(3)×U(1) at a scale of order 10^{17}GeV by the vev of the adjoint Higgs, and then to SU(3)×SU(2)×U(1) at a scale of order 10^{10}-10^{11}GeV by the vev of a fundamental Higgs. The SU(3)×SU(2)×U(1) symmetry is then broken down to SU(3)×U(1) at a scale of order 10^3GeV due to radiative corrections. This model gives us a good prediction for the value of $\sin^2\theta_w$. Although the precise value of $\sin^2\theta_w$ depends on the scale of breaking(v) of the SU(3)×SU(3)×U(1) symmetry, it is relatively insensitive to this scale, and for the range $10^7 m_w < v < 10^{10} m_w$, it varies between .206 and .220. (Note that this is the allowed range of values of v for the Peccei-Quinn symmetry breaking.)

The unified gauge coupling constant at the grand unification scale turns out to be rather large in this model. In fact for most of the otherwise allowed ranges of values of the parameters of the theory, the gauge coupling constant reaches the strong coupling constant before the unification scale. This can be prevented by suitably adjusting the parameters of the theory. However, the constraint of perturbative unification puts a strong restriction on the addition of any more light particles to this model, since this increases the value of α_{GUT}.

There are, however, several questions which remain to be studied, the most important of which is the spontaneous breakdown of the $SU(2)^{weak}\times U(1)$ symmetry. A detailed renormalization group program is needed to study this. The cosmological domain wall problem due to the presence of the exact discrete symmetry group, which is a subgroup of the Peccei-Quinn symmetry, still exists in this model. The solution of this problem may lie in the inflationary model of the early universe[12], if the reheating temperature after inflation is below the Peccei-Quinn phase transition, but still not too much below it, so as to produce enough heavy fermions and/or higgses whose decay may produce the observed baryon to photon ratio of the universe. The possibility of combining the scenario of inflationary universe with the model developed in this paper is under investigation. We also need a detailed study of the cosmological baryon production in this model.

Finally I wish to comment on a possible alteration of this model. As we have seen, the light particle content of the theory is almost uniquely constrained to be that of the minimal model proposed in Sec.II, if we want the gauge coupling to be small at the unification scale. We have also seen that even within the context of the minimal model, the vev of σ needs to be one or two orders of magnitude higher than the $SU(3)\times SU(3)\times U(1)$ breaking scale, in order to ensure the smallness of α_{GUT}. While this can be done by keeping the coupling constant β_2 in the superpotential (1) small, and the mass parameter M_3 large, an alternative is to completely discard the σ, R and $\tilde{R}$ fields, and produce masses of order 10^{12}-10^{13}GeV for the $Q_{20,s}$ and the H fields by coupling them to the fields Φ and Φ_0. This needs small coupling constants of order 10^{-4}-10^{-5}, but this is not too unnatural, since even in the standard Weinberg-Salam model we have such small Yukawa couplings.

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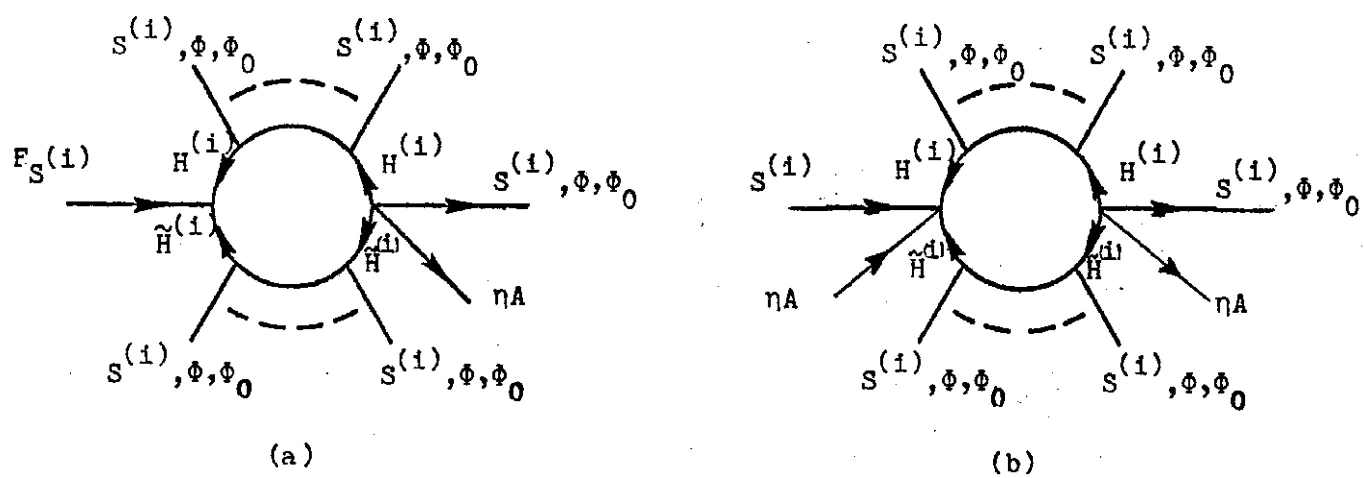
FIGURE CAPTION

Fig.1: Contribution to $f^{(i)}$ [Fig.(a)] and $g^{(i)}$ [Fig.(b)] in one loop order.



$F_{S^{(i)}}$
$S^{(i)}, \Phi, \Phi_0$
$H^{(i)}$
$\tilde{H}^{(i)}$
ηA
(a)
$S^{(i)}$
$S^{(i)}, \Phi, \Phi_0$
$H^{(i)}$
$\tilde{H}^{(i)}$
ηA
(b)

Fig.1